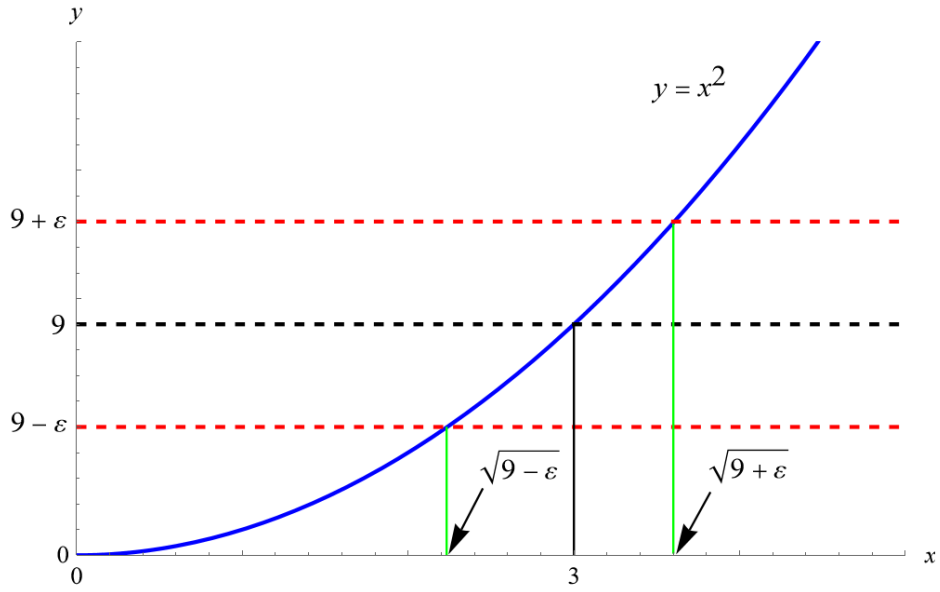
Exercise 34

Verify, by a geometric argument, that the largest possible choice of δ for showing that $\lim_{x\to 3} x^2 = 9$ is $\delta = \sqrt{9+\varepsilon} - 3$.

Solution

Graph the function x^2 versus x.



There seems to be a choice to select $3-\delta$ as $\sqrt{9-\varepsilon}$ or $3+\delta$ as $\sqrt{9+\varepsilon}$. However, because the graph of x^2 curves upward as x increases, the distance from 3 to $\sqrt{9-\varepsilon}$ is larger than the distance from 3 to $\sqrt{9+\varepsilon}$. We select the smaller distance so that the y-values remain between $9-\varepsilon$ and $9+\varepsilon$ as the x-values go between $3-\delta$ and $3+\delta$.

$$3+\delta = \sqrt{9+\varepsilon}$$

Solving for δ,

$$\delta = \sqrt{9+\varepsilon} - 3.$$